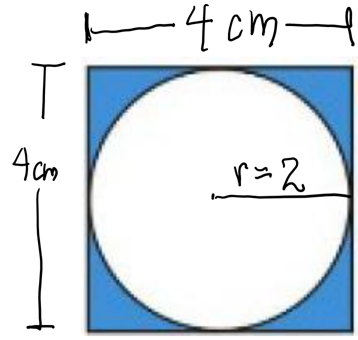
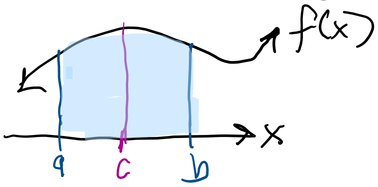


Do: find area of the shaded region given:

$$A_{SHADED} = A_{\square} - A_{\bigcirc}$$
$$= (4)(4) - \pi \cdot 2^2$$
$$= (16 - 4\pi)\ cm^2$$

Know how to add integrals (areas under curve):

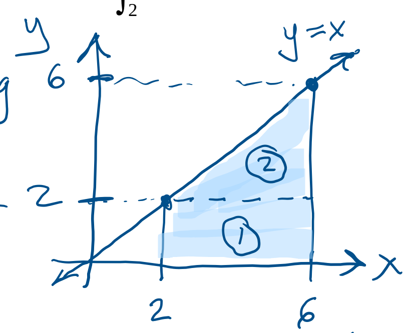


$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

ex. sketch then determine area under the curves: $\int_2^6 x\, dx$ (y=x) $\int_2^6 2\, dx$ (y=2)

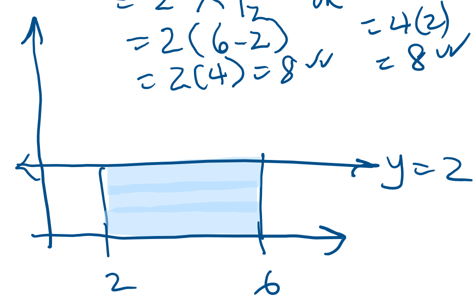
$$\int_2^6 2\,dx = 2\int_2^6 dx = 2\,x\Big|_2^6 = 2(6-2) = 2(4) = 8 \checkmark$$

OR $A_{\square} = bh = 4(2) = 8 \checkmark$



area using standard shapes

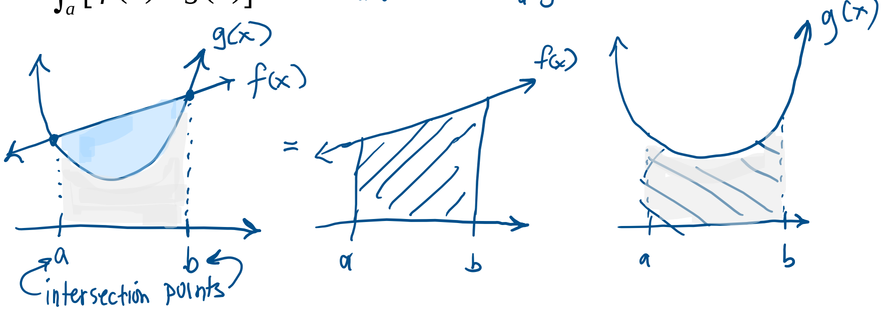
$$\int_2^6 x\,dx = A_1 + A_2$$
$$= 4(2) + \tfrac{1}{2}(4)(4)$$
$$= 8 + 8$$
$$= 16 \checkmark$$



$$\int_2^6 x\,dx = \tfrac{1}{2}x^2\Big|_2^6$$
$$= \tfrac{1}{2}(36-4) = \tfrac{1}{2}(32) = 16\checkmark$$

Finding the Area Bound by Multiple Graphs:
(enclosed)

$$\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)dx - \int_a^b g(x)\,dx$$



Step 1: determine which function has larger y-values

Step 2: determine bounds → a, b by finding the functions' intersection points

Find intersection points by setting functions equal to each other

ex. find the area of the region **bound** by the graphs $f(x) = 2x+1,\ g(x) = x^2+1$

get intersection points:

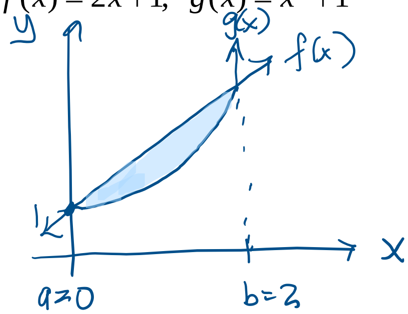
$f(x) = g(x)$

$2x+1 = x^2+1$ solve for x

$0 = x^2 - 2x$

$0 = x(x-2)$

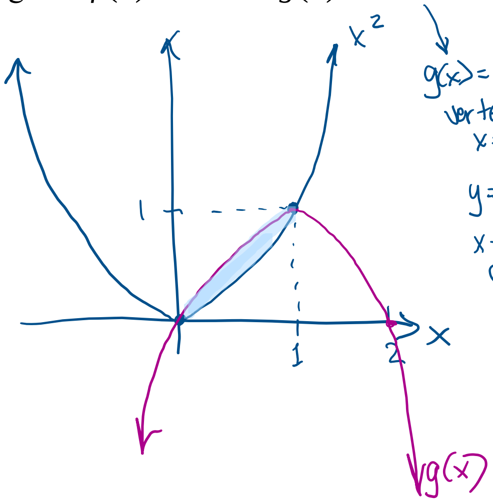
$x_a = 0 \quad x_b = 2$



$$A = \int_0^2 [f(x) - g(x)]\,dx$$
$$= \int_0^2 \left(2x+1 - (x^2+1)\right) dx$$
$$= \int_0^2 (2x + \cancel{1} - x^2 - \cancel{1})\,dx$$
$$= \int_0^2 (2x - x^2)\,dx$$
$$= \left(x^2 - \frac{1}{3}x^3\right)\Big|_0^2$$
$$= 4 - \frac{1}{3}(8)$$
$$= 4\cdot\frac{3}{3} - \frac{8}{3} = \frac{12-8}{3} = \boxed{\frac{4}{3}}$$

Do: given $f(x) = x^2$ and $g(x) = 2x - x^2$ determine the intersection points; sketch on the same plane

can do either 1st



$g(x) = -x^2 + 2x$

vertex: $(1, 1)$

$x = -\frac{b}{2a} = \frac{-2}{2(-1)} = 1$

$y = g(1) = -1 + 2 = 1$

x-int:

$0 = 2x - x^2$

$0 = x(2 - x)$

$x = 0$, 2

$x^2 = 2x - x^2$

$2x^2 - 2x = 0$

$2x(x - 1) = 0$

$x_a = 0$, $x_b = 1$

ex. find area of the enclosed region

$$\int_0^1 g(x) - f(x)\,dx$$

$$= \int_0^1 (2x - x^2 - x^2)\,dx$$

$$= \int_0^1 (2x - 2x^2)\,dx$$

$$= 2\int_0^1 (x - x^2)\,dx$$

$$= 2\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Bigg|_0^1 = \boxed{\frac{1}{3}}$$

Do: finish solving

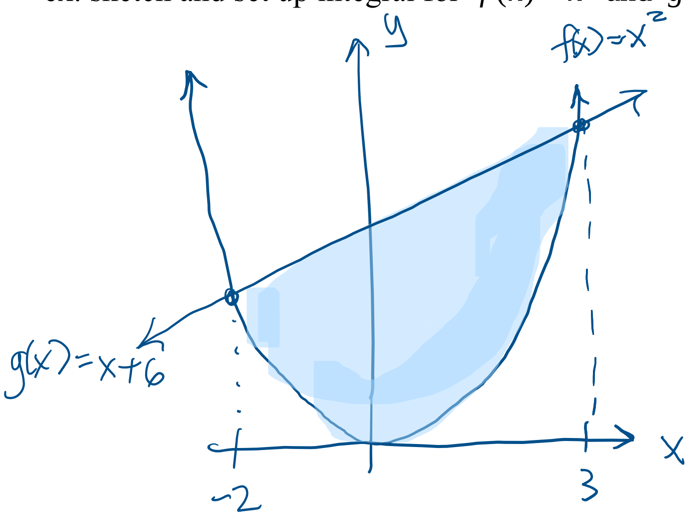
$$= 2\left(\frac{3}{3}\cdot\frac{1}{2} - \frac{1}{3}\cdot\frac{2}{2}\right)$$

$$= \cancel{2}\left(\frac{3-2}{\cancel{2}\,3}\right) = \frac{1}{3}$$

Set Up Integrals for Area Between Two Curves

ex. sketch and set up integral for $f(x)=x^2$ and $g(x)=x+6$

get intersection points



$x^2 = x+6$

$x^2 - x - 6 = 0$

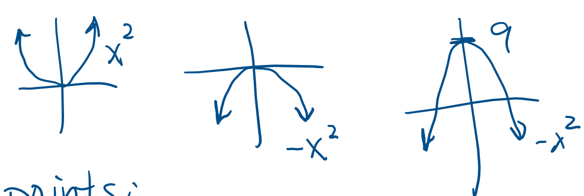
$(x+2)(x-3) = 0$

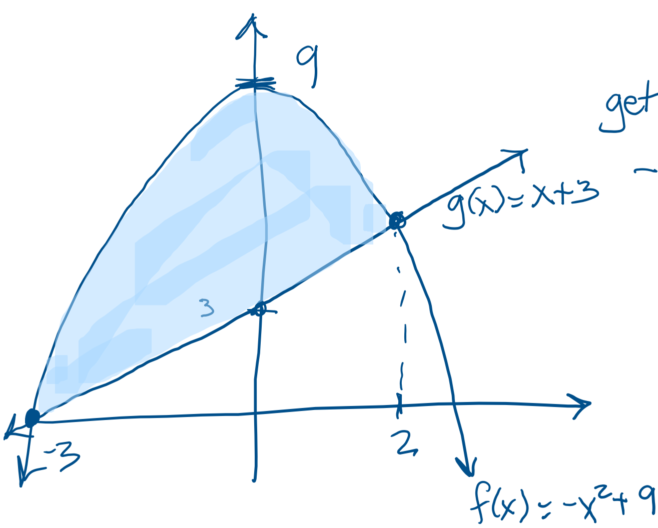
$x_a = -2 \quad x_b = 3$

$g(x) \geq f(x)$

$\int_{-2}^{3} (x+6-x^2)\,dx$

ex. sketch and set up integral for $f(x)=-x^2+9$ and $g(x)=x+3$





get intersection points:

$-x^2+9 = x+3$

$0 = x^2+x-6$

$0 = (x+3)(x-2)$

$x=-3 \quad x=2$

$g(x)=x+3$

get x-int:

$0 = x+3$

$x=-3$

$f(x) \geq g(x)$

$\int_{-3}^{2} (\overbrace{-x^2+9}^{f(x)} - \overbrace{(x+3)}^{g(x)})\,dx$

Recall: Areas involving Even and Odd Functions

Odd Functions' graphs have ___origin___ symmetry because $f(-x) = -f(x)$

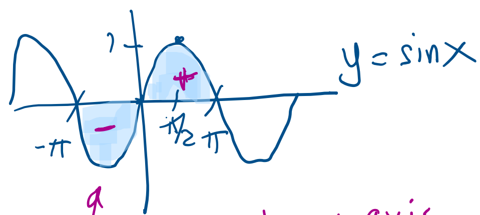
sinx
tanx
x^3

ex. $\int_{-\pi}^{\pi} \sin x \, dx = 0$

odd

$\int_{-a}^{a} f(x) = 0$

odd



areas under x-axis are negative

Even Functions' graphs have ___y axis___ symmetry because $f(-x) = f(x)$

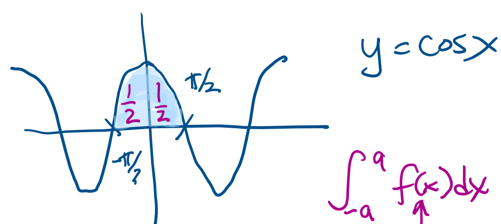
cosx
x^2

ex. $\int_{-\pi/2}^{\pi/2} \cos x \, dx = 2\int_0^{\pi/2} \cos x \, dx$

$= 2 \sin x \Big|_0^{\pi/2}$

$= 2(\sin \frac{\pi}{2} - \sin 0)$

$= 2(1 - 0) = \boxed{2}$



$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

even